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Fractal Dimension of Supergranulation

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Abstract: **Fractal dimension is a valuable mathematical tool to quantify the complexity of geometrical structures. In the context of solar supergranulation, the network of convective cells observed on the solar photosphere, fractal dimension can reveal the dependence of the cell shape complexity on solar rotation, phase, magnetic activity level, and thereby provide insight into the underlying dynamics. This information can be useful for comparing observations with models. In particular, study of supergranular fractal dimension can shed light on the turbulence of the magneto-convective process that generates the magnetic structures. Here we briefly review recent research on this topic in the last decade.**

Keywords: **Fractal dimension, mathematical tool.**

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1. INTRODUCTION

Supergranulation is a cellular pattern found on the solar surface, caused by convection in the Sun . Discovered by A.B. Hart via velocity Dopplergrams in 1950, supergranulation was early on understood to be a convective phenomenon, though its precise origin was not known. Today we understand them as surface manifestations of the sub-photospheric convective currents transporting energy outward from the deeper radiative layers. The convective outer layer has a thickness about 30% of the radius of the Sun. Lying just below the Solar photosphere, it is associated with three network scales of convective eddies: viz., granulation, about 1000 km in size and having a lifetime typically 8-10 min; supergranulation, about 30,000 km in size and having a lifetime typically about a day; and mesogranulation, the status of whose existence is still under debate (Rieutord et al., 2000). Supergranules are observed in the high photosphere as large convection eddies An eddy here refers to a current (of plasma) whose direction of motion differ from that of the principal current. Typically, eddies involve circular motion. Turbulence is an unstable pattern of eddies. In a convective eddy, there is a central upflowing current, a horizontal flow diverging from the centre of the cell and a peripheral downward flow at the cell boundary. The supergranular horizontal currents are thought to sweep magnetic fields from declining active regions into the boundaries of the cell, where they generate the heating which produces the chromospheric network. The chromospheric network thus is largely co-spatial with supergranulation and has a similar lifetime. Supergranular cell evolution involves fields appearing at the center, being transported to the boundary, where they eventually dissipate (Schrijver, Hagenaar, & Title 1997; Hagenaar et al. 1999). Magnetic activity can be expected to affect network cell size (Chandrashekar 1961).In accordance with this, Singh and Bappu (1981) noted a smaller cell size of the Calcium II K cell at solar maxima. That larger cells live longer has also been noted (Singh et al. 1994). The chromospheric network study in the Ca II K and He core spectral lines shows that cell size depends on solar latitude, with approximate North/South symmetry and minima around 20^0N and 20^0S (Raju, Srikanth & Singh 1998). Active cells in the neighborhood of a plage tend to live longer than those in the quiescent regions., which may be attributed to magnetic confinement. An early estimate on size of 26 megameter (Mm) based on horizontal flow is due to Hart (1956), Cells with average lifetime of 20 h and diameter of 35 Mm were reported by November (1994). However, a scatter of sizes with a large number of small cells with mean diameters in the range of 10 Mm to 20 Mm has been reported by various authors (Hagenaar, Schrijver, and Title 1997; Berrilli et al. 1999; and Srikanth, Singh, and Raju, 2000; De Rosa and Toomre, 2004). We may attribute the differences in these reported size values to difference in data analysis methods and in selection of spatial filters. The distribution of supergranular lifetimes is found to be exponential by Del Moro et al. (2004) and De Rosa and Toomre (2004). The latter report that 45% of the supergranules form or get destroyed via merging or fragmentation events, while 20% either spawn off or merge with smaller objects, whereas just 35% arise and dissappear without interacting with neighboring structures. While Hagenaar, Schrijver, and Title (1997), Schrijver, Hagenaar, and Title (1997) and DeRosa & Toomre (2004) report an agreement of supergranulation size distribution with a Voronoi (right-skewed) distribution, an agreement with a rigid-sphere tight-packing model is found by Berrilli et al. (2004).

2. BACKGROUND AND MOTIVATION

Fractal dimension mathematically quantifies the shape complexity of certain geometrical objects, which can be used as a guide to underlying dynamics, and for comparing models with observations. In particular, it can throw light on the turbulence of the sub-photospheric magnetoconvective processes which produce the photospheric magnetic structures.

Quite simply, a set is called a fractal provided it evinces self similarity. That is, it can be split into fragments such that each fragment is approximately a diminished version of the whole. Roudier & Muller (1987) applied fractal analysis to solar granulation, using the perimeter and area data of granules observed from the Pic Du Midi observatory. They report a fractal dimension $D = 1.25$ for smaller granules (diameter $d \leq 1''.37$) and $D \approx 2$ for larger ones.

Turbulent convection dominates the motion solar photospheric velocity field, manifesting in the two major scales noted above: viz. granulation and supergranulation.

Supergranulation is discernible even far from the solar disk center in Dopplergrams (Hathaway et al., 2000), whereas it is barely visible in the Ca II H and K intensity images. Meunier, Tkaczuk, and Roudier (2007) obtained observation supporting the view that a small intensity difference between the boundary and center of the cell would correspond to a temperature contrast of 0.8 – 2.8 K, which, it turns out are consistent with supergranulation having a convective origin.

Indirect observation of supergranulation is possible via tracking of the proper motions of magnetic bright points or other such surface features (Leighton, 1964; Rimmele and Schröter, 1989; Shine, Simon, and Hurlburt, 2000; Rieutord et al., 2007). Since supergranulation was first observed (Hart, 1956; Leighton, Noyes, and Simon, 1962), various explanations of its origin have been proposed. As one possibility, He I or He II ionization in the solar sub-photospheric convective layers causes a local minimum in the radial adiabatic temperature gradient, favoring convective cells having horizontal spans comparable to the ionization zone depth (Simon and Leighton, 1964). As another possibility, spatial ordering of convective cells of smaller scale may be responsible for supergranulation. For instance, mesogranulation scale cells have been seen to form in 2-D simulations, even when insufficiently to reach the ionization zone (Ploner, Solanki, and Gadun, 2000). That supergranulation is a product of collective nonlinear interaction among granules has been suggested (Rieutord et al.,2000; Roudier et al. 2003). On basis of a numerical study of a model for n-body with diffusion-limited aggregation, Crouch, Charbonneau, and Thibault (2007) argued for an oigin of supergranulation by advection of magnetic field elements due to granular eddies**.** This work also supports a negative correlation between magnetic activity level and size of supergranular cell size, an effect which was observed by Meunier, Roudier, and Tkaczuk (2007). Here several methods of local helioseismology have been used to study sup-photospheric supergranular flows (Kosovichev and Duvall, 1997; Duvall and Gizon, 2000;Woodard, 2002; Zhao and Kosovichev, 2003; Braun, Birch, and Lindsey, 2004; Gizon and Birch, 2005). Information about horizontal flow and its divergence have been derived via f-mode time – distance helioseismology by Duvall and Gizon (2000). This same data reveals that supergranulation ranges in 10-40 Mm in size with a mean diameter of about 27 Mm. From autocorrelations derived with up to a 6-day and with a study with longer time lag, a wavelike behavior of supergranulation was discovered (Beck and Duvall, 2001; Gizon, Duvall, and Schou, 2003), which is suggestive of supergranulation being an instance of traveling-wave convection. The wavelike pattern had a lifetime of about 2 days (Gizon, Duvall, and Schou, 2003).

The study of the properties of supergranulation and thereby the underlying convective dynamics remains an active area of research.

3. DISCUSSION

A more complete study of the supergranular network requires a closer look at the interaction among adjacent cells and in specific, the cellular configuration in a given region.

- The arrangement of objects or particles in a structure within a region can be analyzed via the pair-correlation function (PCF), Used mainly in statistical mechanics, pair correlation function, this quantity g (r) characterizes how particle number density depends on the distance r from a reference particle.
- The function gives the probability density for finding a particle at distance r from the center of a given particle.. It can be computed as follows:
- Select a sufficiently small dr, but is statistically significant for counting.
- Fix the range of r that is relevant to the problem:

1. Count the number of particles in a spherical shell of thickness dr at distance r about each given particle.
2. Divide this total count by the number N of particles in your reference set (perhaps all your particles).
3. This number is divided by $4\pi r^2 dr$, which is the shell volume.
4. Normalize this quantity by dividing it by the particle number density.

- Doppler technique used to determine only one component of velocity, viz. line-of-sight velocity-- produeed by supergranular motions, whose amplitude far exceeds that of photospheric motions at a larger-scale velocities, which originally led to the discovery of supergranulation (Hart 1956 and Leighton et al. 1962).
- The technique of tracking tracers in the solar photosphere yields both components of the surface velocity. Historically, Carrington (1859) deduced solar differential rotation by tracking sunspot motion over the solar disk.
- Helioseismological techniques have been grouped into 2 classes: global and local helioseismology. The former involves oscillation mode frequency measurements and matching it with solar seismic models. For a review of methods and results, see Christensen-Dalsgaard (2002). It produces a 2-D map useful to derive the dependence of solar properties on (unsigned) latitude and radius. Global helioseismology was used to infer the angular velocity in the interior of the Sun (Schou et al. 1998, Thompson et al. 2003) and the tachocline, the transition layer at 0.69R that separates the differential rotation in the convection zone (Gizon, Birch & Spruit 2004) and the rigidly-rotating radiative interior. This is perhaps the locus for the solar dynamo. Local helioseismology, employing f-modes and Fourier segmentation in 2-D and 3-D (2-D with third dimension being time), derives maps for the horizontal divergence and a full velocity vector for the flow velocity field at about 1 Mm depth.

However, local helioseismology (Zhao 2004) is known to have room for improvement, and there is a lot of progress in this regard. Owing to the thinness of the photospheric layer (0.04 % times the solar radius), the horizantal predominates the large-scale velocity fields here, for which the tracer-tracking techniques suffice and especially local correlation should be sufficient for mapping the behaviour of such velocities.

LCT, originally intended for the elimination of the seeing-generated distortions in sequences of images (November 1986) and thereafter in order to map granular motion in a white-light image series (November & Simon 1988), is based the idea of the best fit of two sequential frames recording a tracked tracers. Each pixel in a given frame is compared with a small corrresponding somewhat displaced correlation window i**n the subsequent frame.** The displacement vector is defined as the vector difference between the two matched sequential windows. By dividing this by the time lag, one obtains the velocity vector for the pair of frames. LCT can be employed for tracking granules and similar features in high-resolution images obtained by different methods of observation (Sobotka et al. 1999, 2000).

4. CONCLUSIONS

Various observational and mathematical tools like fractal dimension, helioseismology and LCT are employed in order to investigate the geometric forms, motions and statistics associated with supergranulation, the large-scale convective cellular network of typical size 30 Mm (Wang & Zirin 1989) and a wide distribution of lifetimes (DeRosa et al., 2000) with a mean lifetime of approximately 20 h (Leighton 1964). Its shape tends to depend on the solar cycle phase (Sykora

1970). Off the center of the solar disc, the supergranular cell can be detected in a Dopplergrams because of its horizontal internal flow component of about 300 ms^{-1} (Hathaway et al. 2002).

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